Bisectors, Medians, and Altitudes

Worksheet

Day 1:

For #1-4, use a ruler to draw the median and the altitude from vertex A. Use different colors, label which line is the median and which is the altitude.

1. A

2. A

3. A

4. A

5. If \( WP \) is an angle bisector, \( m\angle HWA = (7s + 16) \), \( m\angle PWA = 15 \), find \( s \), and \( m\angle HWA \).

   \( s = \) 

   \( m\angle HWA = \) 

6. Find a counterexample to the statement:

   An altitude and an angle bisector of a triangle are never the same segment

7. If \( MS \) is an altitude of \( \triangle MNQ \), \( m\angle 1 = 3x + 11 \), and \( m\angle 2 = 7x + 9 \). Find \( x \) and \( m\angle 2 \).

   Label the figure. Show all work.

   \( x = \) 

   \( m\angle 2 = \)
8. If $\overline{MS}$ is a median of $\triangle MNQ$, $QS = 3z - 14$, $SN = 2z + 1$, and $m\angle MSQ = 7z + 1$, find $z$. Is $\overline{MS}$ also an altitude of $\triangle MNQ$? Explain. Label the figure. Show all work.

   $z = \underline{\phantom{10000}}$

   Is $\overline{MS}$ also an altitude of $\triangle MNQ$?

9. If $\overline{WP}$ is a perpendicular bisector, $m\angle WHA = (8q + 17)$, $m\angle HWP = (10 + q)$, $AP = 6r + 4$, and $PH = 22 + 3r$, find $r$, $q$ and $m\angle HWP$. Label the figure. Show all work.

   $r = \underline{\phantom{10000}}$

   $q = \underline{\phantom{10000}}$

   $m\angle HWP = \underline{\phantom{10000}}$

10. A batter hits the ball, it lands an equal distance from the 1st and 3rd base foul lines. It is also equidistant from 1st and 2nd base. Using this information, indicate on the picture where the ball is. Draw and label the two special segments that were used to locate the ball.

   Hint 1: If the ball is equidistant from 1st and 3rd base foul lines, use the ___________________ (special segment) to determine the balls location.

   Hint 2: If the ball is equidistant from 1st and 2nd base, use the ___________________ (special segment) to determine the balls location.
11. Explain the similarities and differences between a perpendicular bisector and a median of a triangle.

12. Orienteering is a competitive sport originating in Sweden. It tests the skills of map reading and cross-country running. Competitors race through an unknown area to find various checkpoints using only a compass and topographical map. On an amateur course, clues are given to locate the first flag. Find the flag given the following two hints:

Hint 1: The flag is as far from the Grand Tower as it is from the park entrance. Use the ________________ (special segment) to determine the flags location.

Hint 2: If you run the shortest distance from Sterns Road to the flag or from Amesbury Road to the flag, you would run the same distance. Use the __________________ (special segment) to determine the flags location.

13. Find the measure of each numbered angle if $\overline{AB} \perp \overline{BC}$

   a. $\angle 1 = \underline{\hspace{2cm}}$  
   
   b. $\angle 2 = \underline{\hspace{2cm}}$  
   
   c. $\angle 3 = \underline{\hspace{2cm}}$  
   
   d. $\angle 4 = \underline{\hspace{2cm}}$  
   
   e. $\angle 5 = \underline{\hspace{2cm}}$  
   
   f. $\angle 6 = \underline{\hspace{2cm}}$  
   
   g. $\angle 7 = \underline{\hspace{2cm}}$  
   
   h. $\angle 8 = \underline{\hspace{2cm}}$
Day 2:  

**Special Segments of a Triangle Activity**

**Materials:**
1 sheet of white paper  
1 sheet of colored paper  
Scissors  
Ruler  
Glue

**Step 1:**
- Fold your colored piece of paper into quarters.
- Fold your white piece of paper into quarters.
- Draw a large, scalene, acute triangle onto one of the quarters of white paper.
- Use that triangle to cut out a total of four congruent triangles on the white paper.
- Label the vertices A, B, and C on the interior of each triangle.

**Step 2:**
- Fold the triangles to create the following special segments:
  - Triangle 1: The perpendicular bisector of side BC  
  - Triangle 2: The angle bisector of ∠A  
  - Triangle 3: The median from ∠A to the opposite side  
  - Triangle 4: The altitude from ∠A to the opposite side

**Step 3:**
- After each triangle has been folded:
  - Using a ruler, trace over the fold  
  - Mark all right angles, congruent segments and congruent angles  
  - Label the top of the colored paper "Special Segments"  
  - Glue the triangles on to the colored paper  
  - Label each with the name of the special segment
Chapter 5 Section 1 - Homework

Bisectors, Medians, and Altitudes

Worksheet

Day 3:

1. Lines \( l, m \) and \( n \) are perpendicular bisectors of \( \triangle PQR \) and meet at \( T \).
   \[ TQ = 2x, \ PT = 3y - 1, \ \text{and} \ TR = 8, \ \text{find} \ x, y, \ \text{and} \ z. \] Label the vertices. Show all work.

   \[ \begin{align*}
   x &= \ldots \\
   y &= \ldots \\
   z &= \ldots
   \end{align*} \]

For \#2-5, \( R(3, 3), S(-1, 6) \) and \( T(1, 8) \) are the vertices of \( \triangle RST \) and \( \overline{RX} \) is a median. Graph the vertices. Use a ruler to draw and label the triangle. Show all work.

2. What are the coordinates of \( X \)?

3. Find the equation of line segment \( RX \).

4. Determine the slope of \( RX \)

5. Is \( RX \) an altitude of \( \triangle RST \)? Explain.

For \#6-9, state whether each sentence is always, sometimes, or never true. Justify your reasoning by drawing examples or counterexamples.

6. The three medians of a triangle intersect at a point inside the triangle.
Chapter 5 Section 1 - Homework

7. The three altitudes of a triangle intersect at a vertex of the triangle.

8. The three angle bisectors of a triangle intersect at a point in the exterior of the triangle.

9. The three perpendicular bisector of a triangle intersect at a point in the exterior of the triangle.

10. The vertices of ΔDEF are D(4, 0), E(-2, 4), and F(0, 6).
     Find the coordinates of the circumcenter of ΔDEF.
     Use a ruler to draw and label the triangle. Show all work.

11. Identify the term that does not belong with the other three. Explain your reasoning.

     orthocenter  point of concurrency  altitude  circumcenter
Day 4:

Points of Concurrency Activity

Materials:
1 sheet of white paper
1 sheet of colored paper
Scissors
Ruler
Glue

Step 1:
• Fold your colored piece of paper into quarters.
• Fold your white piece of paper into quarters.
• Draw a large, scalene, acute triangle onto one of the quarters of white paper.
• Use that triangle to cut out a total of four congruent triangles on the white paper.
• Label the vertices A, B, and C on the interior of each triangle.

Step 2:
• Fold the triangles to create the following special segments:
  o Triangle 1: Three perpendicular bisectors - one on each side of the triangle.
    Label the point where they intersect “Circumcenter”
  o Triangle 2: Three angle bisectors - one from each angle in the triangle.
    Label the point where they intersect “Incenter”
  o Triangle 3: Three medians - one on each side of the triangle.
    Label the point where they intersect “Centroid”
  o Triangle 4: Three altitude - One on each side of the triangle.
    Label the point where they intersect “Orthocenter”
Step 3:
- After each triangle has been folded:
  - Using a ruler, trace over the folds
  - Mark all right angles, congruent segments and congruent angles
  - Label the top of the colored paper “Points of Concurrency”
  - Glue the triangles on to the colored paper
  - Label each with the name of the point of concurrency
  - Under the triangles (or wherever you have room), list the special characteristics of each point of concurrency
    Example:
    - The Circumcenter is equidistant from all three vertices of the triangle.
    - The Incenter is equidistant from all three sides of the triangle.
    - The Centroid is two-thirds the distance from each vertex to the midpoint of the opposite side.
    - The Centroid is the balancing point of the triangle.
  - On the triangles, in a different color, mark the following special characteristics of each point of concurrency
    - The Circumcenter is equidistant from all three vertices of the triangle.
    - The Incenter is equidistant from all three sides of the triangle.
    - The Centroid is two-thirds the distance from each vertex to the midpoint of the opposite side.