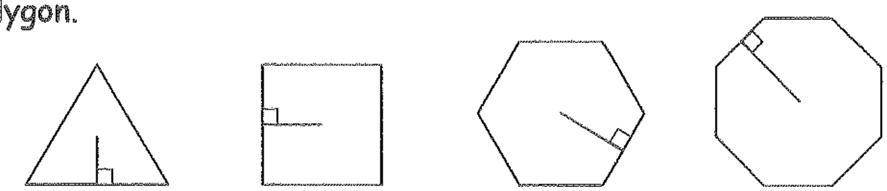
Chapter 11 Section 3 - Day 2 Name Key

Area of Regular Polygons & Circles

Notes

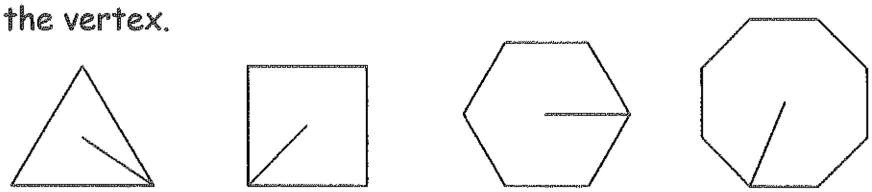
Apothem:

A segment drawn from the center of a regular polygon perpendicular to the side of the polygon.



Radius:

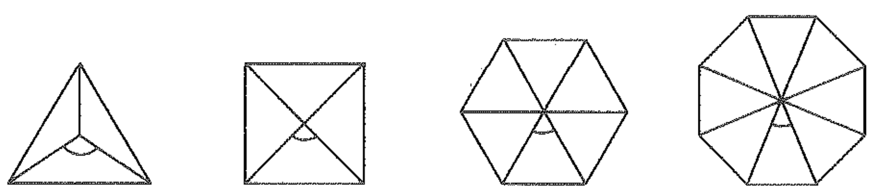
A segment drawn from the center of a regular polygon to the side of the polygon at the vertex.



Central Angle:

The angle with its vertex at the center of the polygon created when 2 adjacent radii are drawn.

$$\text{Central Angle} = \frac{360^\circ}{\#\text{ of sides}}$$

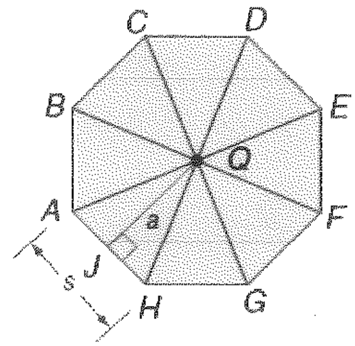


$= \frac{360^\circ}{3}$ $= \frac{360^\circ}{4}$ $= \frac{360^\circ}{6}$ $= \frac{360^\circ}{8}$

$= 120^\circ$ $= 90^\circ$ $= 60^\circ$ $= 45^\circ$

Area of Regular Polygons:

The radii of a polygon are congruent, so if all radii are drawn, they create congruent isosceles triangles. The area of the polygon can be found by adding the areas of all the triangles created by the radii.



Example 1:

The area of $\Delta QAH = (\frac{1}{2})(b)(h)$

where:

$s \longrightarrow$ (side of the polygon) is the base of the triangle
$a \longrightarrow$ (the apothem) is the height of the triangle

So: $\Delta QAH = (\frac{1}{2})(s)(a) \longrightarrow$ area of each triangle

There are 8 triangles in an octagon so the area of the octagon is:

$$A = (8)(\tfrac{1}{2})(s)(a)$$

The perimeter of the octagon is: $P = (8)(s)$

So: $A = (\frac{1}{2})(a)(8)(s)$
Substitute: $A = (\frac{1}{2})(a)(P)$

Formula for the Area of any Regular Polygon : $A = (\frac{1}{2})(a)(P)$

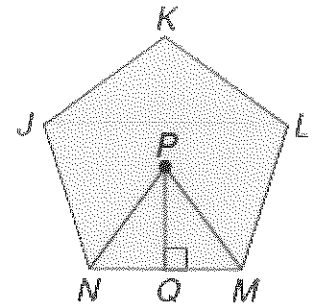
Area of a Circle:

Formula for the Area of a Circle: $A = \pi r^2$

Name_______________________

Example 2:

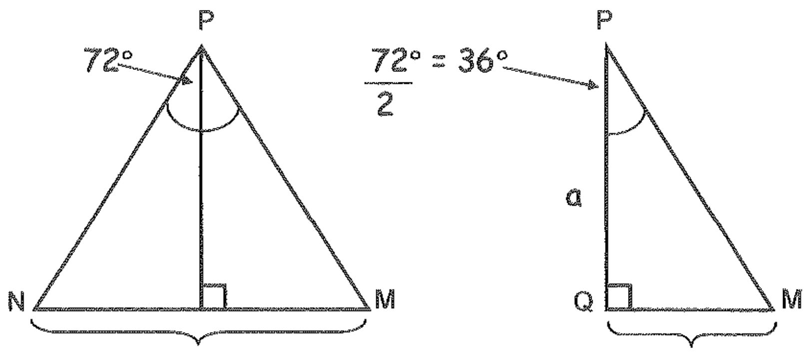
Find the area of a regular pentagon with a perimeter of 40 centimeters.



1) Find the Central Angle: $\frac{360°}{5} = 72°$

2) Given the Perimeter (P): P = 40

3) Need to find the Apothem (a):



$\frac{40}{5} = 8\text{cm}$ ← (Pentagon)

$\frac{8}{2} = 4\text{cm}$

So: $\tan(36) = \frac{4}{a}$

$a = \frac{4}{\tan(36)}$

a = 5.5 cm

4) Find the Area of the Pentagon:

$A = (\frac{1}{2})(a)(P)$

$A = (\frac{1}{2})(5.5)(40)$

$A = 110\text{cm}^2$

Name_____________________________

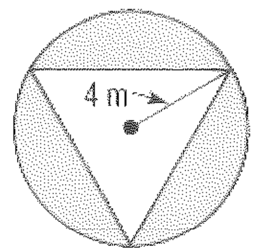
Example 3:

Find the area of the shaded region. Assume that the triangle is equilateral.

1) Find the Area of the Circle: $r = 4$

Area = $16\pi m$ $A = \pi r^2$

$= 16\pi$



2) Find the Area of the Triangle:

Central Angle = $120°$ $\frac{360}{3}$

Radius = 4m

Apothem = 2m

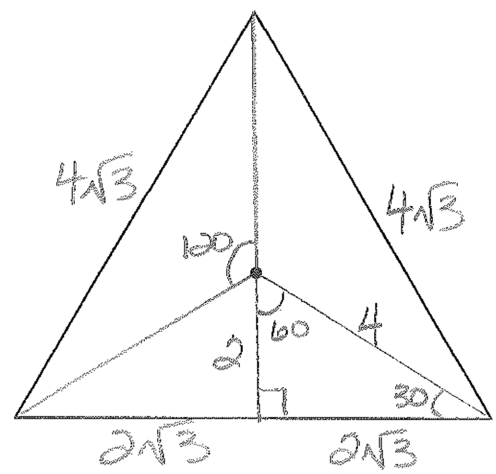
Perimeter = $12\sqrt{3}$m

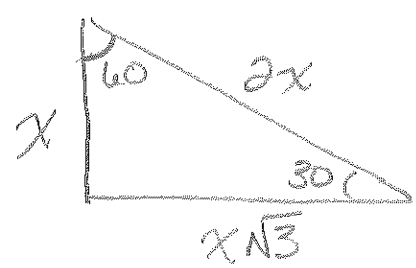
Area = $12\sqrt{3}$m

$A = \frac{1}{2}aP$

$= \frac{1}{2}(2)(12\sqrt{3})$

$= 12\sqrt{3}$





3) Subtract the Areas to Find the Area of the Shaded Region:

Area = $29.5 m^2$ Shaded = $A_{circle} - A_{triangle}$

$= 16\pi - 12\sqrt{3}$

$= 50.265 - 20.784$

$= \boxed{29.481}$